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MATHEMATICAL, PHYSICAL & ENGINEERING SCIENCES

Weak Interactions in the Early Universe: Is the Universe Open?

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Phil. Trans. R. Soc. Lond. A 1980 **296**, 415-429
doi: 10.1098/rsta.1980.0186

Phil. Trans. R. Soc. Lond. A **296**, 415–429 (1980) [415]
Printed in Great Britain

Weak interactions in the early Universe: is the Universe open?

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Big-bang nucleosynthesis *per se* cannot decide if the universe is open or not. Present gauge theories of elementary particles favour, on several grounds, low values (much less than unity) of the universal lepton numbers. These values, in the context of nucleosynthesis compatibility, still suggest an open Universe ($\Omega \leqslant 0.2$).

1. Introduction

We review here the interrelations between (*a*) the nucleosynthesis of the light elements (D, ^{3}He, ^{4}He, ^{7}Li), (*b*) the universal baryonic density, and (*c*) some recent developments in particle physics. In particular we address three important questions:

(1) Does the nucleosynthesis of the light nuclei imply a low density Universe ($\Omega \leqslant 0.1$) (where Ω is the ratio of the present density ρ_b to the closure density ρ_c; for $H = 75$ km s^{-1} Mpc^{-1}, $\rho_c = 1.07 \times 10^{-29}$ g cm^{-3})?

(2) In case that, from astronomical observations, Ω *does* turn out to be large ($\Omega > 0.2$), do we have to throw away the hot big bang on account of nucleosynthesis?

(3) What would be the implications for the unification models of physical interactions, by gauge theories, of such large Ω?

First, we review the physics of the situation and introduce a few useful parameters. There is good evidence nowadays that leptons come in pairs. The electron pair is composed of the electron (e) and the electron-neutrino (ν_e); the muonic pair is composed of the mu meson (μ) and the muon-neutrino (ν_μ). Recently a new pair has been added: the tauonic pair with the tau (τ) and the tau-neutrino (ν_τ). How many such pairs exist?

To each pair is attached a leptonic number which, it seems, is conserved separately in each reaction. It should be said, however, that the absolute conservation of each individual number is nowadays a matter of some debate. For example, the muon number is defined as

$$L_\mu = [n(\mu^-) - n(\mu^+) + n(\nu_\mu) - n(\bar{\nu}_\mu)]/n(\gamma). \tag{1}$$

The term $n(\gamma)$ is the denominator is the number of photons in the relic radiation (*ca.* 400 cm^{-3}). The particle physicists do not introduce this extra term. The astrophysicists like to include it because this way they obtain an L_μ that does not change in a spatial expansion.

Nowadays the muon population is zero; but what do we know about the muon-neutrino population of our Universe? And do we have muon symmetry [$n(\nu_\mu) = n(\bar{\nu}_\mu)$]?

The hot big-bang theory provides a partial answer to this question. In the very early moments ($t \ll 1$ s), thermal equilibrium between weak interactions ensured that the neutrino populations were given by the appropriate Fermi–Dirac expression:

$$n(\nu_\mu) = \frac{1}{2\pi^2}\left(\frac{kT_\nu}{\hbar c}\right)^3 \int_0^\infty \frac{x^2\,\mathrm{d}x}{1+\exp\,(x-\xi_\mu)}. \tag{2}$$

27-2

Here $x = pc/kT$, while $\xi_\mu = \mu(\nu_\mu)/kT$ is the ratio of the chemical potential $\mu(\nu_\mu)$ of the ν_μ to their thermal energy. Around $kT \approx 1$ MeV the Universe became too cold for the equilibration to be maintained; the ν_μ were then decoupled from the rest of the world. The later expansion has affected their population solely through the presence of T_ν in (2). Because of the electron-positron annihilation phase taking place when $kT \leqslant 1$ MeV, the temperature of the photons T_γ is now slightly larger than the temperature of the neutrinos T_ν (of all kinds). The two temperatures are related by simple multiplicity factors

$$(T_\gamma/T_\nu)^3 = \tfrac{11}{4} \qquad (3)$$

Nowadays $T_\gamma \approx 3$ K. Thus $T_\nu \approx 2$K. This is the value to be used in (2).

The early thermal equilibrium phase guarantees that the sum of the chemical potentials of the ν_μ and the $\bar{\nu}_\mu$ should be zero ($\mu(\nu_\mu) = -\mu(\bar{\nu}_\mu)$), but does not guarantee that they are individually of null value.

As a result, the population of antineutrinos ($n(\nu_\mu)$), which is given by an equation such as (2) but with ξ_μ replaced by ($-\xi_\mu$), will be equal to the population $n(\nu_\mu)$, if and only if $\xi_\mu = 0$. Thus an asymmetry in the leptonic world will always be related to a non-zero chemical potential (we use ξ_μ rather than $\mu(\nu_\mu)$ since the former is conserved during the expansion) (Weinberg 1972). The quantitative relation (given by Beaudet & Goret 1976) is

$$L_\mu = \frac{2}{3}\left(\frac{kT_\nu}{\hbar c}\right)^3 (\xi_\mu + \pi^{-2}\xi_\mu^3)/n(\gamma); \qquad (4)$$

with the value $T_\nu = 2$ K we get

$$L_\mu \approx 0.25\,(\xi_\mu + 0.1\xi_\mu^3). \qquad (5)$$

For our later discussion we introduce also a neutrino charge density as

$$J_\mu = n(\gamma)\,L_\mu. \qquad (6)$$

The effect of ξ_μ on the energy density of the muon neutrino is given by

$$\rho(\nu_\mu) + \rho(\bar{\nu}_\mu) = (aT_\nu^4/c^2)\,\{\tfrac{7}{8} + \tfrac{15}{4}\pi^{-2}\xi_\mu^2 + \tfrac{15}{8}\pi^{-4}\xi_\mu^4\}. \qquad (7)$$

Here a is Boltzmann's constant. The first term ($\frac{7}{8}$) is present because of the mere existence of the muonic-lepton pair. The terms in ξ_μ^2 and ξ_μ^4 manifest the effect of the asymmetry described before.

Expressions analogous to (1)–(7) are defined for all three pairs of leptons known nowadays (e, μ, τ).

The number of such pairs and their hypothetical asymmetry have a direct effect on the time scale for expansion, in the early Universe, through the expression

$$\tau_{\text{exp}}^{-1} \equiv \dot{R}/R \approx \surd(\tfrac{8}{3}\pi G\rho), \qquad (8)$$

where R is the expansion parameter (for instance the mean distance between two galaxies) and ρ is the energy density of everything. During the period of nucleosynthesis, the baryonic energy density is negligible. The major contribution comes from relativistic particles whose energy density varies as the fourth power of the temperature:

$$\rho c^2 = aT_\gamma^4\{1 + \tfrac{7}{4} + \tfrac{7}{8}n_1 + \text{terms in } \xi_e, \xi_\mu, \xi_\tau, \ldots\}, \qquad (9)$$

where the number 1 represents the photons; the $\frac{7}{4}$ comes from the positrons and electrons; the $\frac{7}{8}$ and terms in ξ represent the neutrinos and their hypothetical asymmetry, n_1 being the number of leptonic pairs. The ellipsis leaves room for undiscovered lepton pairs.

Clearly the existence of asymmetries and the existence of yet undiscovered neutrino types would both shorten the time scale for expansion and thus directly affect the yield of big-bang nucleosynthesis. The effect is double. First, the decoupling of the weak interactions, with consequent lack of reaction equilibrium, would take place at an earlier time, and higher temperature, T_{dec}, when more neutrons are still around (the n/p ratio is higher). Secondly, the period between weak interaction decoupling and neutron capture by protons is shortened. Fewer neutrons will have time to decay and a larger helium yield will result.

In this game the electron-lepton pair plays another role which essentially stems from the facts that (1) the electron is stable (while the muon and tauon are not) and (2) the rest mass of the electron (0.511 MeV) is quite similar to the neutron–proton mass difference ($\Delta M(\text{n}-\text{p}) \approx 1.3$ MeV).

The quantity L_e is defined as in (1). While $n(\text{e}^+)$ is zero, $n(\text{e}^-)$ is not. Because of charge neutrality, we have $n(e^-) = n(\text{p}) \approx 0.85\, n(\text{B})$, where $n(\text{B})$ is the present baryon (neutron and proton) number and the factor 0.85 represents the neutrons dwelling in stable nuclei (mostly helium).

The fact that our Universe is not symmetric in baryons, $L_{\text{B}} = [n(\text{B}) - n(\overline{\text{B}})]/n(\gamma) \approx 10^{-9}$, ensures that it is not symmetric in electrons either:

$$L_e \approx 0.85 L_{\text{B}} + (n(\nu_e) - n(\overline{\nu}_e))/n(\gamma). \tag{10}$$

We shall introduce also r_e, the ratio of the electron leptonic asymmetry to baryon asymmetry:

$$r_e \equiv L_e/L_{\text{B}} \approx 0.85 + (n(\nu_e) - n(\overline{\nu}_e))/n(\text{B}). \tag{11}$$

For our discussion, the effect of the first term on nucleosynthetic yield is negligible and thus will no longer be considered.

Above $T \approx 10^{10}$ K ($kT > m_e c^2$), the reactions between n, p, e^+, e^-, ν_e and $\overline{\nu}_e$ are in equilibrium:

$$\begin{aligned} \text{n} + \text{e}^+ &\rightleftharpoons \text{p} + \overline{\nu}_e; \\ \text{p} + \text{e}^- &\rightleftharpoons \text{n} + \nu_e. \end{aligned} \tag{12}$$

The neutron: proton ratio is then given by

$$\text{n/p} = \exp - \left[\frac{\Delta M(\text{n}-\text{p})}{kT} + \xi_e\right]. \tag{13}$$

The role of ξ_e will be understood through the following example. Assume that $\xi_e > 0$. Thus $L_e > 0$ (equation 4) and $n(\nu_e) > n(\overline{\nu}_e)$ (equation 1). In the reactions (12) the equilibrium (compared with the case $\xi_e = 0$) will be shifted toward more protons and less neutrons, as seen in (13).

The yield of helium will be largely governed by the value of n/p in (13) at the decoupling temperature T_{dec}. It is of interest to note the respective roles played by (*a*) the neutron–proton mass difference, (*b*) the asymmetry parameter of the electron-neutrinos and (*c*) the decoupling temperature (related to the strength of the weak interactions and to the number and asymmetries of *all* types of neutrinos). These quantities are expected to be strongly related in the formalism of a unified theory of the physical interactions.

2. Nucleosynthetic yields

To describe the effect of neutrino existence and asymmetries on nucleosynthetic yields, we shall use as parameters Ω (the baryon density), and an effective ξ'_μ defined in the following equation (compare with (9)):

$$\rho c^2 = aT_\nu^4\{1+\tfrac{7}{2}+\tfrac{15}{4}\pi^{-2}\xi'^2_\mu+\tfrac{15}{8}\pi^{-4}\xi'^4_\mu\}. \tag{14}$$

The case $\xi'_\mu = 0$ applies to a world with only two neutrinos (ν_e, ν_μ) and no asymmetries. The discovery of the third neutrino (ν_τ) corresponds, if it is massless, to the case $\xi'_\mu = 1.44$. A fourth one would correspond to $\xi'_\mu = 1.96$. In table 1, the number N of symmetric lepton pairs is given as a function of ξ'_μ. (Quite generally, the ξ'_μ can describe any physical element which would increase the universal density and vary with the fourth power of T_ν). An equivalent L'_μ can be defined via (4).)

Table 1. The number of symmetric pairs of massless neutrinos and antineutrinos ($n(\nu_\mu) = n(\bar{\nu}_\mu)$) as a function of the equivalent ξ'_μ defined in (14)

N	ξ'_μ
2 (e, μ)	0
3 (e, μ, τ)	1.44
4 (e, μ, τ, ...)	1.16
5	2.33
6	2.62
10	3.69
50	6.23
5000	21.6

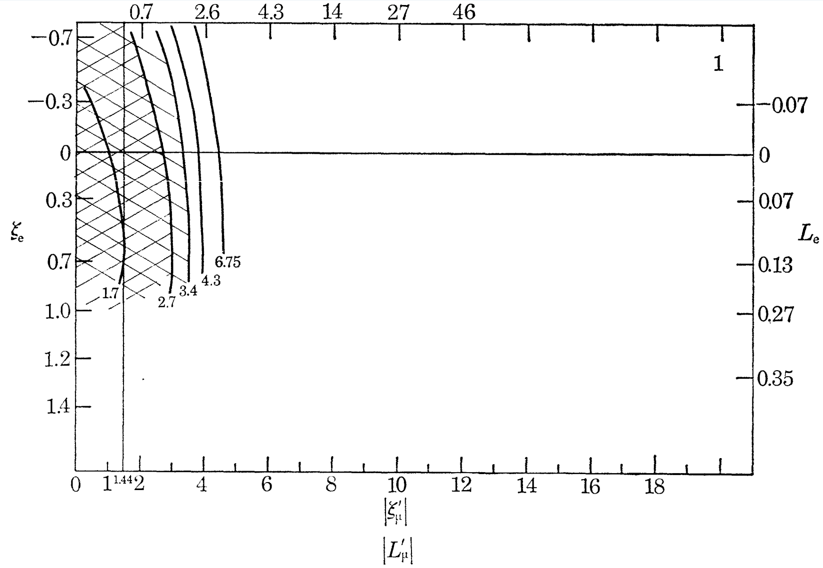
From astronomical observations some limits can be obtained on the expansion rate of the Universe (from the absence of asserted effects on the deceleration parameter). The issue is far from being settled because of complications of various kinds; however, it is probably safe to set $\Omega < 1.5$ (Weinberg 1972) corresponding to $|\xi_\mu| \lesssim 50$ (Beaudet & Yahil 1977) or $N \lesssim 1.4\times 10^5$, quite a large number indeed. This limit also applies to $|\xi_e|$.

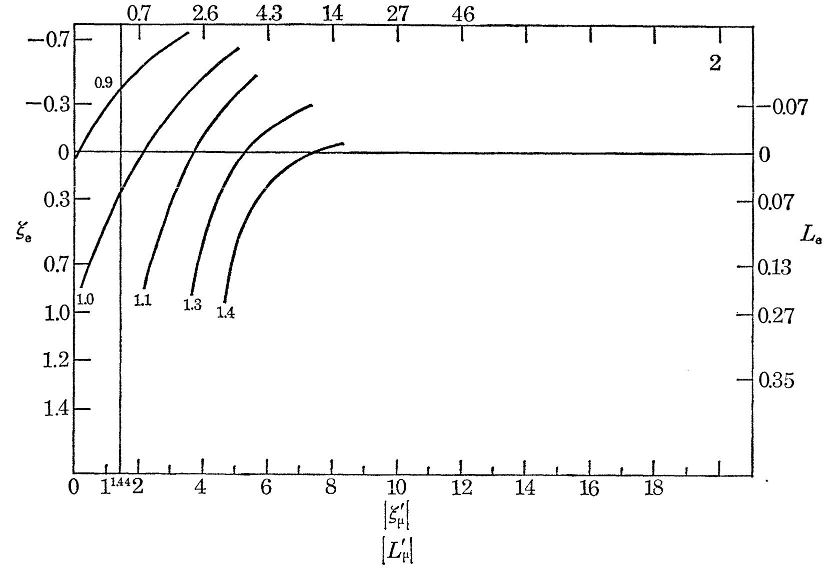
In figures 1–8 we present the isoyield curves of the four nuclides D, ^{3}He, ^{4}He and ^{7}Li as a function of ξ_e (or L_e) and $|\xi'_\mu|$ (or $|L'_\mu|$) (the sign of ξ'_μ does not matter (cf. equation 7)), for two values of Ω, 0.21 and 0.04. (All yields in this paper are obtained from the program of Beaudet & Yahil (1977).)

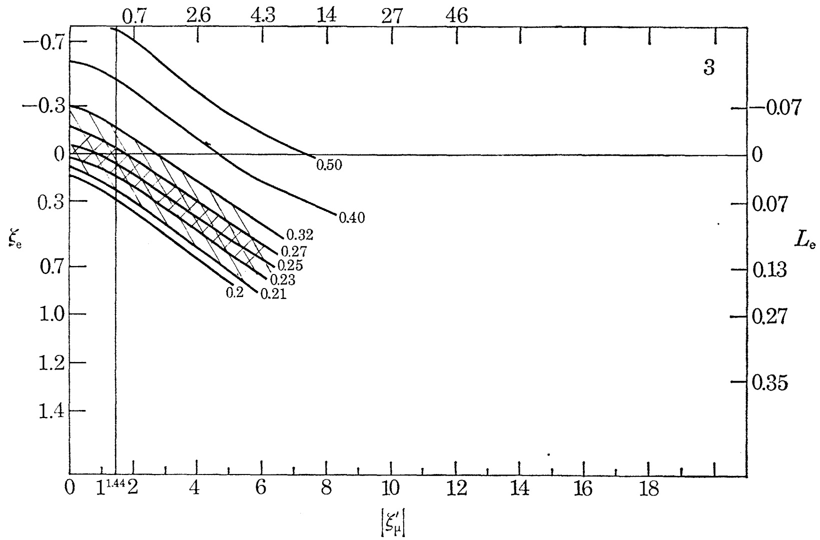
The curves of ^{4}He are easily understood via (8), (13) and (14) since most of the neutrons end up as ^{4}He (Peebles 1971). Increasing ξ'_μ will decrease the time scale of expansion. The weak interaction will run out of equilibrium sooner, and at a higher T_{dec}. The fraction of neutron decaying before nucleosynthesis will also be smaller. The consequent increase in ^{4}He can be matched by increasing ξ_e (and thus decreasing the n/p ratio at equilibrium).

Other nuclides are not as simply treated, as they involve several formation and destruction reactions. The yield of D is essentially independent of ξ_e. ^{3}He and ^{7}Li have more complex relation to these parameters (^{7}Li has two zones: one corresponds to the formation of ^{7}Be, the other to ^{7}Li itself).

Because of these different behaviours, the four nuclei could, in principle, be used to determine the values of the three parameters Ω, ξ_e and $|\xi'_\mu|$ (this was the main motivation for the present study). Unfortunately the behaviours of D, ^{3}He and ^{7}Li do not differ enough for this goal to be yet accessible. Rather accurate determinations of the abundances would be required to take full use of the existence of the four nuclides. Nevertheless, some interesting conclusions can already be reached.







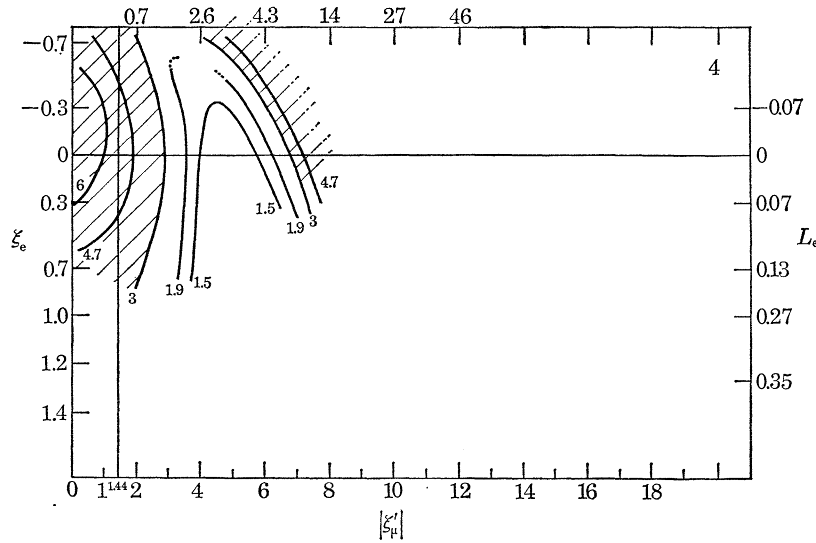


FIGURE 1. Isoyield curve for D in big-bang nucleosynthesis with $\rho_b = 5.4\times10^{-31}$ g cm^{-3} ($\Omega \approx 0.05$). The left ordinate is the chemical potential of the ν_e: $\xi_e = \mu(\nu_e)/kT_\nu$; the right ordinate is the corresponding electronic lepton number L_e. The abcissa is the equivalent muonic chemical potential ξ'_μ defined in (14). The numbers on the curves are number density ratios.

FIGURE 2. Isoyield curve for ^{3}He in big-bang nucleosynthesis with $\rho_b = 5.4\times10^{-31}$ g cm^{-3} ($\Omega \approx 0.05$).

FIGURE 3. Isoyield curve for ^{4}He in big-bang nucleosynthesis with $\rho_b = 5.4\times10^{-31}$ g cm^{-3} ($\Omega \approx 0.05$).

FIGURE 4. Isoyield curve for ^{7}Li in big-bang nucleosynthesis with $\rho_b = 5.4\times10^{-31}$ g cm^{-3} ($\Omega \approx 0.05$).

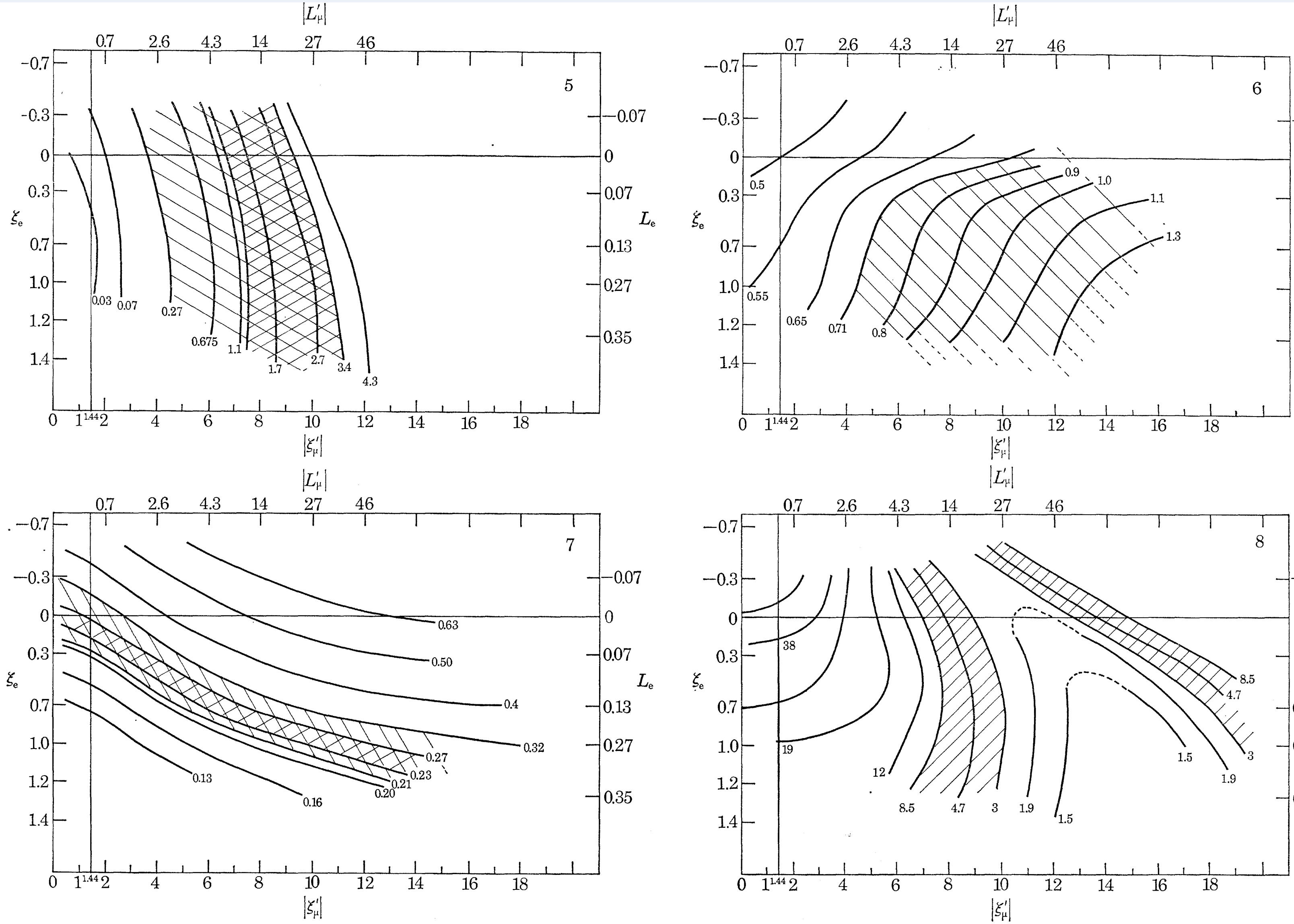


FIGURE 5. Isoyield curve for D in big-bang nucleosynthesis with $\rho_b = 2.26\times10^{-30}$ g cm^{-3} ($\Omega \approx 0.2$).

FIGURE 6. Isoyield curve for ^{3}He in big-bang nucleosynthesis with $\rho_b = 2.26\times10^{-30}$ g cm^{-3} ($\Omega \approx 0.2$).

FIGURE 7. Isoyield curve for ^{4}He in big-bang nucleosynthesis with $\rho_b = 2.26\times10^{-30}$ g cm^{-3} ($\Omega \approx 0.2$).

FIGURE 8. Isoyield curve for ^{7}Li in big-bang nucleosynthesis with $\rho_b = 2.26\times10^{-30}$ g cm^{-3} ($\Omega \approx 0.2$).

3. Abundance values and uncertainties: how to play singly and doubly sure

We are faced here with two different problems: (1) to determine correctly the abundances and their uncertainties, and (2) to extrapolate backwards in time and identify properly the big-bang contributions. The second problem involves possible production by later mechanisms as well as destruction by astration during galactic life.

We shall work simultaneously at two levels. The first is more or less conventional: from available data and galactic evolution model we make a choice of the likely values of abundances, with 'reasonable' uncertainties. This would be the 'one-sigma (1σ) approach' in standard deviation theory.

Table 2. Abundances of D, ^{3}He, ^{4}He, ^{7}Li to be used in comparison with calculations of big-bang nucleosynthesis

(The observed values are extrapolated to pregalactic period.)

	1σ	2σ
$n(\mathrm{D})/n(\mathrm{H})$	$(1.0–2.5)\times10^{-5}$	$(0.3–3.0)\times10^{-5}$
$n(^3\mathrm{He})/n(\mathrm{H})$	$(1.0–2.0)\times10^{-5}$	$< 3\times10^{-5}$
$n(^4\mathrm{He})/n(\mathrm{H})$	0.074–0.092	0.068–0.115
Y	0.23–0.27	0.21–0.32
$n(^7\mathrm{Li})/n(\mathrm{H})$	$(0.3–0.8)\times10^{-9}$	$< 4\times10^{-9}$

The left column gives ranges of values incorporating 'reasonable' values of the uncertainties (or equivalent 1σ uncertainties). At right, more remote possibilities of errors are included and the range of uncertainties are extended (an equivalent 2σ set).

In view of the importance of the cosmological implications, however, it seems advisable to define a second set of uncertainties, characterized by a much higher degree of scepticism: 'How far wrong can we after all expect to be?' This is a sort of 'two-sigma (2σ)' approach.

Conclusions based on this '2σ' set will clearly be weaker in content but stronger in credibility than conclusions obtained from the '1σ' set. Our sets are given in table 2. Some comments are given here.

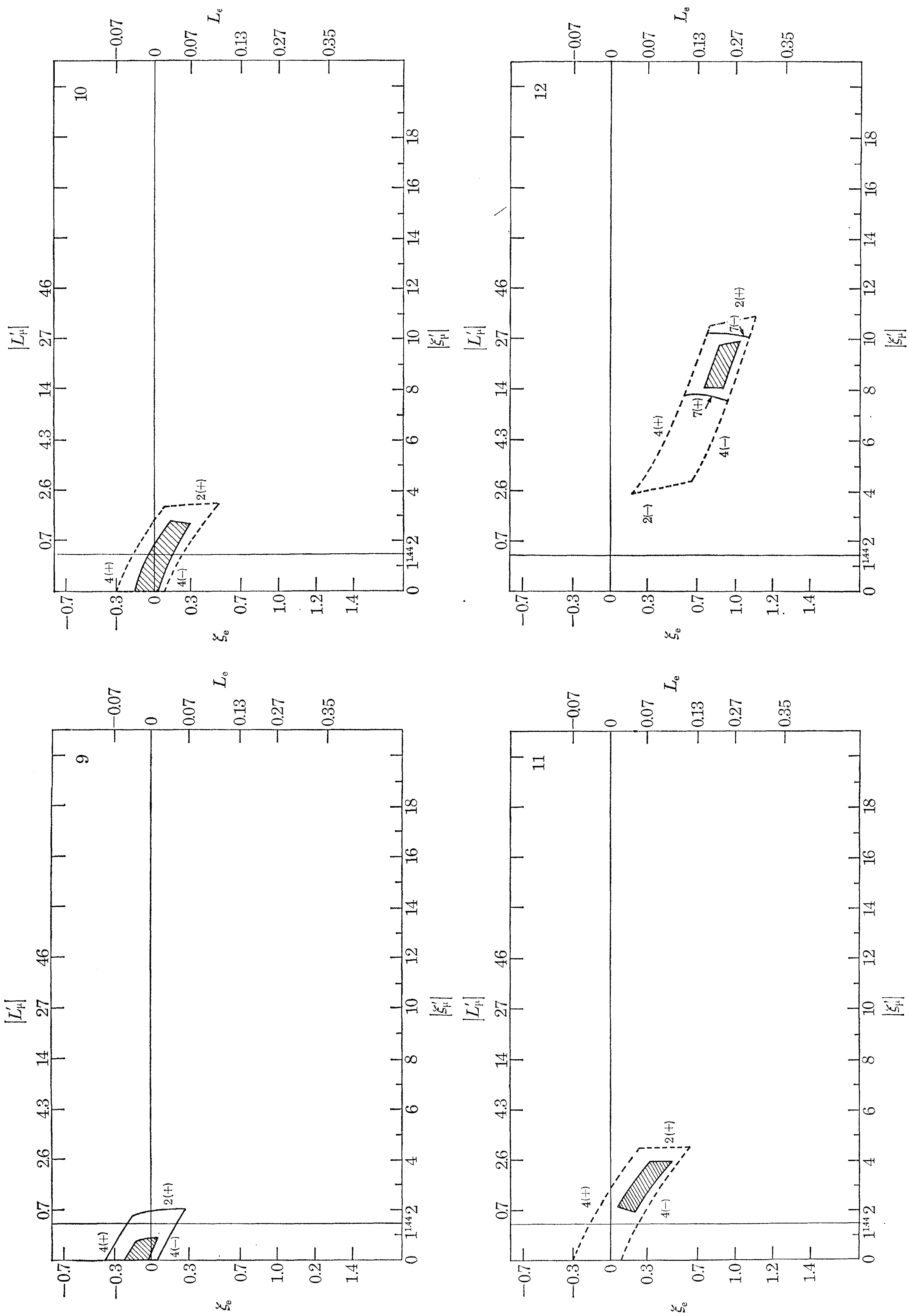
Deuterium

The 1σ of D includes most of the observed data (York & Rogerson 1976; Vidal-Madjar *et al.* 1977; Laurent 1978). We assume no major source of D after the big bang and suppose rather modest amount of astration $n(\mathrm{D})_{\mathrm{b.b.}} \leqslant 2n(D)_{\mathrm{obs}}$. This last view is partly supported by the fact that matter exchange between galaxies and intergalactic space is gaining some weight (Reeves & Meyer 1978; Oort 1970; Hunt & Sciama 1972; Hunt 1975).

The 2σ includes data whose relevance to our problem has been questioned, and incorporate the possibility of larger astration effects. It still assumes that D is predominantly of big bang origin. This, of course, is still questionable but, in our opinion, less and less so (see Epstein *et al.* (1976) for a review of the physics of the situation).

Helium-4

The origin of ^{4}He has been discussed by many authors (Danziger 1970; Burbidge & Burbidge 1975; Peimbert 1975; Smith 1975). The big-bang origin is hardly in doubt. The stellar contribution (discussed by Peimbert *et al.* 1978) is likely to be small ($\Delta Y \leqslant 0.02$).



FIGURES 9–12. For description see opposite.

PHILOSOPHICAL TRANSACTIONS OF THE ROYAL SOCIETY A MATHEMATICAL, PHYSICAL & ENGINEERING SCIENCES

Helium-3

A search for ^{3}He in interstellar space (Rood *et al.* 1979) has not given any positive result. Their most interesting piece of information is perhaps the upper limit of ^{3}He/H from W49 (^{3}He/H $< 2\times10^{-5}$).

In the Solar System, ^{3}He/^{4}He ratios have been measured in meteorites and in the solar wind. These data are partly 'contaminated' by the burning of $D+p \rightarrow {}^3He$ in the early Sun. Geiss & Reeves (1972) have discussed these problems and given $10^{-5} \leqslant {}^3\mathrm{He/H} \leqslant 2\times10^{-5}$ in the early Solar System.

However, ^{3}He can also be generated in stars. In the 2σ set we simply require the big-bang not to overproduce ^{3}He.

Lithium-7

A fraction (from 0.2 to 0.5) of the ^{7}Li is generated by galactic cosmic rays (Reeves & Meyer 1978). Astration should be quite similar for ^{7}Li as for D. But as a function of Ω the effect goes in the opposite direction (this was cleverly used by Mathews & Viola 1979). Again stellar generation is possible: our 2σ set requires no overproduction.

Compatibility?

The task is to look for compatibility areas in the ξ_e, ξ'_μ plane for fixed values of ρ_b (or Ω), by using both the 1σ and 2σ sets.

At $\rho_b \leqslant 2\times10^{-31}\,\mathrm{g\,cm^{-3}}$ ($\Omega \leqslant 0.01$†), no compatibility area exists even with the extended set. This value turns out to be the lower limit on ρ_b obtained from galaxy counts (Peebles 1964). We should not overlook the importance of this coherence between nucleosynthesis and astronomical observation.

The situation for $\rho_b = 4\times10^{-31}\,\mathrm{g\,cm^{-3}}$ ($\Omega = 0.04$) is described in figure 9. The two sets of uncertainties define two areas of compatibility. The centre of the black cross is the point $\xi_e = 0$

† With $H = 75\,\mathrm{km\,s^{-1}\,Mpc^{-1}}$.

FIGURE 9. Areas of compatibility of the big bang nucleosynthesis yields with the observed abundances at $\rho_b = 4.0\times10^{-31}\,\mathrm{g\,cm^{-3}}$ ($\Omega = 0.04$); (*a*) according to the 1σ set (inner hatched region); (*b*) according to the 2σ set (larger region). The coordinates are those of figure 1. The centre of the cross is, for $\xi_e = 0$, $\xi'_\mu = 1.44$. The curves including figures (2, 3, 4 or 7) are isoyields (respectively of D, ^{3}He, ^{4}He and ^{7}Li). Close to those that correspond to a limiting value of 2σ set stands a + or a − sign, indicating the region of overproduction or underproduction of the relevant element. Other isoyield curves are also shown and the corresponding number density is then indicated. Notice that if the ν_τ turns out to be massless (or $m_{\nu_\tau} < 1$ MeV) the big-bang yields can only be compatible with the 2σ set (and are excluded in the 1σ set by *both* D and ^{7}Li abundances).

FIGURE 10. Areas of compatibility of the big-bang nucleosynthesis yields with the observed abundances at $\rho_b = 5.4\times10^{-31}\,\mathrm{g\,cm^{-3}}$ ($\Omega = 0.05$): (*a*) according to the 1σ set (inner hatched region); (*b*) according to the 2σ set (larger region). The conventions are the same as in figure 9. This density agrees very well with $\xi_e \ll 1$, and up to 16 extra massless fermionic degrees of freedom.

FIGURE 11. Areas of compatibility of the big-bang nucleosynthesis yields with the observed abundances at $\rho_b = 7.21\times10^{-31}\,\mathrm{g\,cm^{-3}}$ ($\Omega = 0.07$): (*a*) according to the 1σ set (inner hatched region); (*b*) according to the 2σ set (larger region). The conventions are the same as in figure 9. This is about the limit density that can accommodate $\xi_e \ll 1$ (and only with the 2σ set).

FIGURE 12. Areas of compatibility of the big-bang nucleosynthesis yields with the observed abundances at $\rho_b = 2.3\times10^{-30}\,\mathrm{g\,cm^{-3}}$ ($\Omega = 0.21$): (*a*) according to the 1σ set (inner hatched region); (*b*) according to the 2σ set (larger region). The conventions are the same as in figure 9. This order of magnitude for Ω might be required by the necessity to bind the clusters. The high ξ'_e might be provided by any kind of extra (expansion accelerating) energy density but ξ_e can no longer be much less than unity.

MATHEMATICAL, PHYSICAL & ENGINEERING SCIENCES
PHILOSOPHICAL TRANSACTIONS OF THE ROYAL SOCIETY A

$\xi'_\mu = 1.44$ (including the ν_τ but no asymmetries). This point is allowed by the $2-\sigma$ set but lies outside of the 1σ set (yielding too much).

At $\rho_b = 5.4\times10^{-31}$ g cm^{-3} ($\Omega = 0.5$, figure 10) the yields are compatible with all requirements. The 1σ set could perhaps accommodate one extra type of neutrino (after the ν_τ) but probably no more. At $\rho_b = 7.2\times10^{-31}$ g cm^{-3} ($\Omega = 0.07$) (figure 11) the 1σ set is incompatible with $\xi_e = 0$, $\xi'_\mu = 0$ or 1.44. The 2σ set admits these points. At $\rho_b = 2.26\times10^{-30}$ g cm^{-3} ($\Omega = 0.2$) (figure 12) even the 2σ set does not admit $\xi_e = 0$ (D is underproduced *and* Li is overproduced). The 1σ set requires $\xi_e \approx 0.7$ and $\xi'_\mu \approx 9$. At $\rho_b = 7.0\times10^{-30}$ g cm^{-3} (figure 13) ($\Omega = 0.7$) we require $\xi_e \approx 1.2$ and $\xi'_\mu \approx 18$.

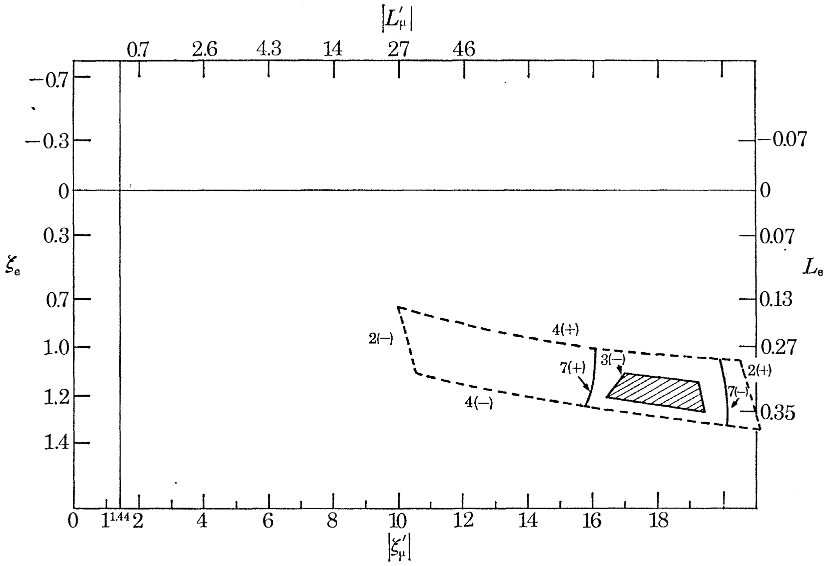


FIGURE 13. Areas of compatibility of the big-bang nucleosynthesis yields with the observed abundances at $\rho_b = 7.2\times10^{-30}$ g cm^{-3} ($\Omega = 0.67$): (*a*) according to the '1σ' set (inner hatched region); (*b*) according to the '2σ' set (larger region). The conventions are the same as in figure 9, the coordinates are those of figure 1. This case serves as an example of the situation around the closure density. The compatibility area is quite far from the standard big bang conditions. It is noticeable that although the 2σ compatibility region is quite large, most of its area is allowed *only* when releasing the limits of the 1σ set for at least two elements.

4. CONCLUSIONS

In summary we can obtain nucleosynthesis compatibility areas at all values of Ω from 0.02 to 1.5 provided we are ready to consider non-zero leptonic numbers, a conclusion already reached by Beaudet & Yahil (1977). The situation is shown graphically in figure 14 where the permissible incursions in the (ξ_e, ξ'_μ) plane are shown in correspondence with the set of uncertainties. Thus, in answer to our first question, the nucleosynthesis of the light elements does *not* necessarily imply a low density Universe.

As discussed before, undiscovered pairs of neutrinos would increase the value of ξ'_μ (14). Let us now consider this possibility, while restricting all pairs to a symmetric state (all $L \ll 1$, $r_e \approx 1$). Based on the figures 9–13, with $\xi_e \ll 1$ we can make the following statements with some confidence: $\Omega < 0.1$, $\xi'_\mu < 2.7$. Therefore the number of still undiscovered lepton pairs is less than five and thus the total number of neutrino degrees of freedom is less than 16. The weaker credibility statement, based on the 1σ set is that $\Omega \leqslant 0.07$, $\xi'_\mu < 1.8$ and there is at most one new lepton pair to be discovered (Steigman *et al.* 1977). For the sake of discussion, let us now consider the possibility

PHILOSOPHICAL TRANSACTIONS OF THE ROYAL SOCIETY A MATHEMATICAL, PHYSICAL & ENGINEERING SCIENCES

that astronomical observations force us to larger values of Ω. We are then forced to forsake the hypothesis $L_e \ll 1$. We discuss two cases:

(*a*) If $\Omega = 0.2$, figure 12 tells us with strong credibility that $L_e > 0.07$ and with weak credibility that it is around 0.2. The ratio r_e is larger than 2×10^7 (*ca.* 6×10^7) and the neutrino charge density (equation 6) J is larger than 30 (*ca.* 100 cm^{-3}).

(*b*) At $\Omega = 0.7$ the situation is as follows: $L_e \geqslant 0.15$ (*ca.* 0.35); $r_e \geqslant 3 \times 10^7$ (*ca.* 6×10^7); $J > 150$ (*ca.* 300 cm^{-3}).

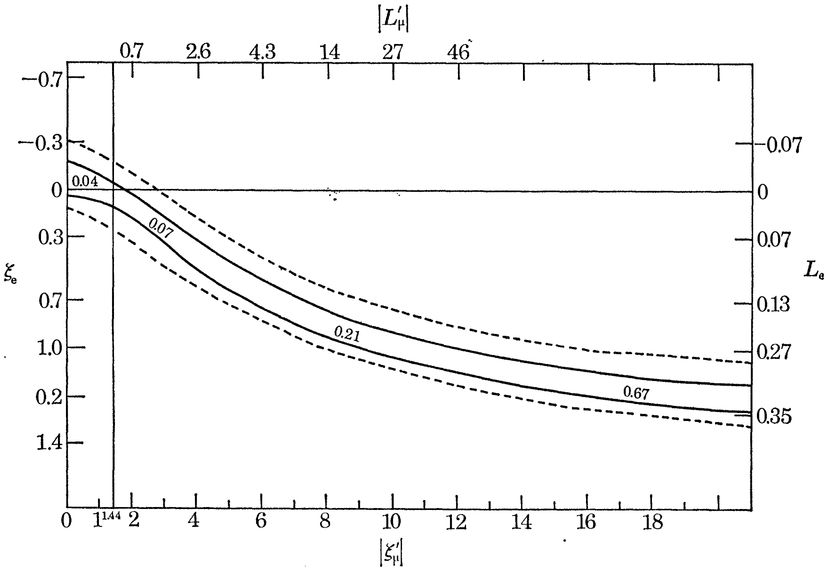


FIGURE 14. The cumulative compatibility regions in the (ξ_e, ξ'_μ) plane for Ω ranging from 0.04 to *ca.* 0.7 appear as a continuous path (the inner path being made of the 1σ compatibility domains); this is due to the 'stability' of the limiting ^4He isoyield curves throughout this range. The allowed region along this path changes with Ω through the D, ^3He, ^7Li requirements. The numbers appearing inside the inner path given an estimate of the required value of Ω along the path. One important conclusion is that we have to forsake the hypothesis $\xi_e \ll 1$ when $\Omega \geqslant 0.1$.

Particle physics

To investigate the implications of the non-zero leptonic numbers in particle physics we shall briefly summarize the present situation. These days witness the triumph of gauge theories, which seem to place within reach the unification of several (if not all) of the elementary interactions.

The Weinberg–Salam–G.I.M. way of unifying weak and electromagnetic interactions (Weinberg 1967; Salam 1968; Glashow, Iliopoulos & Maiani 1970) is experimentally well founded, each experiment making it, year after year, stronger. Nevertheless other possible schemes exist, which are compatible with the experimental results. Some of them require explicitly the existence of new, massive (or not), paired (or not) neutrinos.

As for the strong interactions, the favourite theory is also a gauge theory known as 'Chromodynamics' (Politzer 1974; Marciano & Pagels 1978). In spite of a greater complexity in extracting experimentally interesting results, it has already met with considerable success, although some fundamental issues remain to be cleared up (especially the problem of quark confinement).

The possibility of escaping the use of gauge theories has been repeatedly advocated (mainly by Bjorken 1978). However, some of the satisfactory results that the theories have had spontaneously come from their typical gauge theoretic shape, in particular from the way they get renormalizable.

Taking into account the detailed structure of these models, renormalizability has led to the necessity of a symmetry between quark pairs and lepton pairs. The validity of the whole scheme was confirmed by the successive discoveries of the $\tau - \nu_\tau$ pair, and of the alleged member of the corresponding quark-doublet: the b-quark found in the upsilon particle. Non-abelian gauge theories also provide a unique way of explaining the asymptotic freedom of the strong interactions (Politzer 1974).

Several 'Grand Unifying' schemes have been presented for these two gauge theories (Weinberg–Salam and Chromodynamics). Among these the Georgi–Glashow SU(5) is an outstanding example (Georgi & Glashow 1974; Georgi *et al.* 1974; Chanowitz *et al.* 1977; Nanopoulos 1978). A natural (but not unavoidable) consequence of putting quarks and leptons on the same footing is the decay of the quarks and consequently of the nucleons. This will eventually lead to the disappearance of matter. This might also explain the appearace of a non-zero baryon number out of a symmetrical big bang (Yoshimura 1978; Dimopoulos & Susskind 1978). In the SU(5) model, the expected proton life time is *ca.* 10^{33} years (Jarlskog & Ynduráin 1979), only a few powers of ten above the present experimental limit. Confirmation of proton decay would be an important step. It would hint on how to unify interactions. Within a few years we should know.

In spite of numerous encouraging points, none of these theories, of course, can be considered as the final word in elementary physics. In fact, the task of making the gravitational interaction join the others might (despite its already gauge theoretic shape) be a tremendous one, although it might also be the only way to get a satisfactory account of these currently better 'understood' interactions.

To end this brief description of the present status in particle physics, we should stress the very interesting behaviour of these theories when high energies are involved. In particular our picture of the very early big bang is profoundly modified by the possible restoration of the broken gauge symmetries (making the weak and electromagnetic interactions appear as complementary, setting, at still higher energies, a unique footing for the strong and electroweak and even, perhaps, a unification with gravitation) (see, for example, Weinberg 1974; Khirznits & Linde 1976; Linde 1979).

As we have seen before, when ξ_e (or L_e) and ξ'_μ are allowed non-zero values, the nucleosynthesis compatibility requirements do not force us to $\Omega < 0.1$ or to a small number of lepton pairs. However, from the theories described above, a few interesting points emerge:

(*a*) The present scheme foresees that although L_e, L_μ, L_τ and B might vary, the number $(B-L)$ should be conserved (where L is the algebraic sum of L_e, L_μ and L_τ). Thus $L_e \ll 1$.

(*b*) Another important fact in this connection is the influence of a leptonic charge density (thus of a neutrino density (equation 6)) on the restoration of the electroweak gauge symmetry. The exact nature of this influence has been a matter of debate (Harrington & Yildiz 1974; Lee & Wick 1974). According to Khirznits & Linde (1976), $j_e \gtrsim 10^2$ cm^{-3} would be a threat for this restoration never to occur.

(*c*) The number of expected lepton pairs is at most three or four as deduced from the calculated fermion masses in the SU(5) model (Nanopoulos & Ross 1978).

A weaker limit on the number leptons is imposed from the necessity to preserve Chromodynamics's asymptotic freedom property, within the alleged quark–lepton symmetry.

Even if the currently favourite models are not unquestionable, the above remarks are not easily escaped. They remarkably point towards a low L_e and thus a low Ω Universe.

PHILOSOPHICAL TRANSACTIONS OF THE ROYAL SOCIETY A MATHEMATICAL, PHYSICAL & ENGINEERING SCIENCES

Astrophysical discussion

As previously mentioned, the purely astrophysical lower limit on Ω is $\Omega \gtrsim 0.02$, while the upper limit is $\Omega \lesssim 1.5$. Discussions of its actual value have been presented at this symposium by Gunn and Fall.

In this respect, one very important problem is that of the missing mass needed to bind the clusters of galaxies. The original version developed by Zwicky in 1933 was based on a comparison between the observed velocities and masses of the member galaxies. The Coma cluster, for instance, has an observed mass of about one-tenth of that required to account for its tight configuration.

This case has been strongly supplemented by the observations of large X-ray-emitting clouds around certain individual galaxies (in particular M87) and clusters of galaxies. These clouds would expand and disappear quickly if they were not gravitationally confined. For Coma, the mass needed to bind the cloud is of the order of the mass necessary to bind the cluster. This extra mass, which could exist in the form of numerous red dwarfs in halos of galaxies, would also help considerably in understanding the large abundance of iron present in the intercluster gas (see, for example, Gorenstein & Tucker 1978). The mass of the clusters would then appear to be large enough to set $\Omega \gtrsim 0.1$).

Another argument for large Ω is based on galaxy counts and on the study of the correlation functions of galaxy positions at large distances. Computer models were used to follow the development of galaxy clusters from an early uniform state to the present state. With $\Omega \approx 1$ the results are n reasonably good agreement with the observations but for $\Omega \leqslant 0.1$ they are unacceptable. The significance of this result, in view of the large uncertainties, has been discussed at length, at this meeting. It is fair to say that no agreement has been reached on this question (see also Aarseth *et al.* 1979; Turner *et al.* 1979).

A referendum between astrophysicists would probably set $0.05 \leqslant \Omega \leqslant 1$ with most people favouring the lower part of the range. A more accurate determination is not yet possible but would become a reality with the very large telescope (Oort 1979).

General discussion

There is an interesting coherence between the view of the elementary particle physicists, the nucleosynthesists and the astronomers.

The successful theories of particle interaction favour (*a*) low (much less than 1) values of leptonic numbers, (*b*) small neutrino charge density and (*c*) at most three or four lepton pairs. The nucleosynthesist finds that, with these constraints, he can account for the abundance of the nuclides D, ^{3}He, ^{4}He and ^{7}Li provided $\Omega < 0.1$ (forgetting for the moment effects such as local density inhomogeneities in the Universe (Olson & Silk 1978). The astronomers generally find the range 0.05–0.1 quite to their taste.

However, it is, in our view, too early to exclude $\Omega \approx 1$. The nucleosynthesist would then be forced to $L_e \approx 1$, together with $\xi'_\mu \approx 20$. Could these values be reconciled with elementary particle physics? This is yet an open problem. If not, big bang nucleosynthesis would find itself in a very difficult situation.

About helium-3

We should like to conclude with a point that is seldom made and that deals with ^{3}He.

The inspection of the value of the ^{3}He yield in the regions of the (ξ_e, ξ'_μ) plane resulting, for each Ω, in appropriate D, ^{4}He and ^{7}Li production, reveals a very restricted range:

$$0.5 \times 10^{-5} \leqslant {}^3\text{He/H} \leqslant 2 \times 10^{-5}.$$

It is more remarkable that this range agrees very well with observations, as discussed in §3.

It is therefore our opinion that this fact can hardly be taken as a chance coincidence and that its (perhaps light) weight should add to the already well documented case in favour of the hot big-bang. A corollary is that the stellar contribution to ^{3}He is at most of the same order as the big bang.

This is therefore the place to stress the urgent need for interstellar detection of ^{3}He, because if an observation in the expected range would be an agreeable event, a *lack* of detection at the $^3\text{He/H} < 3 \times 10^{-6}$ level could be one of the strongest cases *against* hot big-bang nucleosynthesis. We are anxiously waiting for these most interesting observations.

This text is based on a paper in preparation by Y. David, H. Reeves & G. Beaudet.

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